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Quantum vacuum fluctuations and the cosmological constant

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Abstract

Zeta function regularization techniques are optimally suited for the calculation of the contribution of fluctuations of the vacuum energy of the quantum fields pervading the universe to the cosmological constant (cc). The order of magnitude calculations of the absolute contributions of all fields is known to lead to a value which is off by over 120 orders, as compared with the results obtained from observational fits, known as the *new cc problem*. This is difficult to solve and many authors still stick to the old problem to try to prove that basically its value is zero with some perturbations thereof leading to the (small) observed result (Burgess C P *et al* 2006 *Preprints* hep-th/0606020, 0510123, Padmanabhan T 2006 *Preprint* gr-qc/0606061, etc). We also address this issue in a somewhat similar way, by considering the *additional* contributions to the cc that may come from the possibly non-trivial topology of space and from specific boundary conditions imposed on braneworld and other seemingly reasonable models that are being considered in the literature (mainly with other purposes too)—kind of a Casimir effect at cosmological scale. If the ground value of the cc would be indeed zero, we would then be left with this perturbative quantity coming from the topology or BCs. We review the status of this approach, in particular the fact that the computed number is of the right order of magnitude (and has the right sign, what is also non-trivial) when compared with the observational value, in some of the aforementioned examples.

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1. Introduction

Regularization and renormalization procedures are essential issues in contemporary physics [1]. Among the different methods, zeta function regularization—obtained by analytic continuation in the complex plane of the zeta function of the relevant physical operator in each case—is the most beautiful. Use of this procedure yields, for instance, the vacuum energy corresponding to a quantum physical system, with constraints of very different nature. The case of moving boundaries seems to present quite severe difficulties, though some promising approach in order to deal with them has been issued recently [2]. Assume the Hamiltonian operator, H, of our quantum system has a spectral decomposition of the form (think as always, as simplest case, in a quantum harmonic oscillator): $\{\lambda_i, \varphi_i\}_{i\in I}$, I being some set of indices (which can be discrete, continuous, mixed, multiple, ...). Then, the quantum vacuum energy is obtained as follows [3]:

$$E/\mu = \sum_{i\in I} \langle \varphi_i, (H/\mu)\varphi_i\rangle = \mathrm{Tr}_\zeta\, H/\mu = \left.\sum_{i\in I} (\lambda_i/\mu)^{-s}\right|_{s=-1} = \zeta_{H/\mu}(-1), \qquad (1)$$

where ζ_A is the zeta function corresponding to an operator A, and the equalities are in the sense of analytic continuation (since, generically, the Hamiltonian operator will not be of the trace class).[2] The formal sum over the eigenvalues is usually ill defined and the last step involves analytic continuation, inherent with the definition of the zeta function itself. Also, an unavoidable renormalization parameter, μ, with dimensions of mass appears in the process, in order to render the eigenvalues of the resulting operator dimensionless, so that the corresponding zeta function can actually be defined. For the lack of space, we shall not discuss those important details here, which are just at the starting point of the whole renormalization procedure. The mathematically simple-looking relations above involve deep physical concepts, no wonder that understanding them took several decades in the recent history of quantum field theory.

2. On the zero-point energy and the Casimir force

In an ordinary QFT, one cannot give a meaning to the *absolute* value of the zero-point energy, and any physically measurable effect comes as an energy *difference* between two situations, such as a quantum field in curved space as compared with the same field in flat space, or one satisfying boundary condition (BC) on some surface as compared with the same in its absence, etc. This difference is the Casimir energy: $E_C = E_0^{BC} - E_0 = \frac{1}{2}(\mathrm{tr}\, H^{BC} - \mathrm{tr}\, H)$. But here a problem appears. Imposing mathematical boundary conditions (BCs) on physical quantum fields turns out to be a highly non-trivial act. This was discussed in detail in a paper by Deutsch and Candelas [5]. These authors quantized them and scalar fields in the region near an arbitrary smooth boundary, and calculated the renormalized vacuum expectation value of the stress–energy tensor, to find out that the energy density diverges as the boundary is approached. Therefore, regularization and renormalization did not seem to cure the problem with infinities in this case and an infinite *physical* energy was obtained if the mathematical BCs were to be fulfilled. However, the authors argued that surfaces have non-zero depth, and its value could be taken as a handy dimensional cut-off in order to regularize the infinities. Just two years after Deutsch and Candelas' work, Symanzik carried out a rigorous analysis of QFT in the presence of boundaries [6]. Prescribing the value of the quantum field on a boundary

[2] The reader should be warned that this ζ-trace is actually no trace in the usual sense. In particular, it is highly non-linear, as often explained by the author [4]. Some colleagues are unaware of this fact, which has lead to serious mistakes and erroneous conclusions published sometimes in top journals.

means using the Schrödinger representation, and Symanzik was able to show rigorously that such representation exists to all orders in the perturbative expansion. He also showed that the field operator being diagonalized in a smooth hypersurface differs from the usual renormalized one by a factor that diverges logarithmically when the distance to the hypersurface goes to zero. This requires a precise limiting procedure and point splitting to be applied. In any case, the issue was proven by him to be perfectly meaningful within the domains of renormalized QFT. In this case the BCs and the hypersurfaces themselves were treated at a pure mathematical level (zero depth) by using Dirac delta functions.

Recently, a new approach to the problem has been postulated [7]. BCs on a field, ϕ, are enforced on a surface, S, by introducing a scalar potential, σ, of Gaussian shape living on and near the surface. When the Gaussian becomes a delta function, the BCs (Dirichlet here) are enforced: the delta-shaped potential kills *all* the modes of ϕ at the surface. For the rest, the quantum system undergoes a full-fledged QFT renormalization, as in the case of Symanzik's approach. The results obtained confirm those of [5] in the several models studied albeit they do not seem to agree with those of [6]. They seem to be also in contradiction with those quoted in the usual textbooks and review articles dealing with the Casimir effect [8], where no infinite energy density when approaching the Casimir plates has been reported.

3. On the topology and curvature of space

The Friedmann–Robertson–Walker (FRW) model, which can be derived as the *only* family of solutions to Einstein's equations compatible with the assumptions of *homogeneity* and *isotropy* of space, is the generally accepted model of the cosmos. But the FRW is a family with a free parameter, k, the curvature, that can be either positive, negative or zero (the flat or Euclidean case). This curvature, or equivalently the curvature radius, R, is not fixed by the theory and should be matched with cosmological observations. Moreover, the FRW model, and Einstein's equations themselves, can only provide local properties, not global ones, so they cannot tell about the overall topology of our world: is it closed or open, finite or infinite? Even being quite clear that it is, in any case, extremely large—and possibly the human species will never reach more than an infinitesimally tiny part of it— the question is very appealing to any (note that this discussion concerns only three-dimensional space curvature and topology, time will not be involved).

3.1. On the curvature

Serious attempts to measure the possible curvature of the space we live in go back to Gauss, who measured the sum of the three angles of a big triangle with vertices on the picks of three far away mountains (Brocken, Inselberg and Hohenhagen). He was looking for evidence that the geometry of space is non-Euclidean. The idea was brilliant, but condemned to failure: one needs a much bigger triangle to try to find the possible non-zero curvature of space. Now cosmologists have recently measured the curvature radius R by using the largest triangle available, namely one with us at one vertex and with the other two on the hot opaque surface of the ionized hydrogen that delimits our visible universe and emits the CMB radiation (some 3–4 $\times 10^5$ years after the big bang) [9]. The CMB maps exhibit hot and cold spots. It can be shown that the characteristic spot angular size corresponds to the first peak of the temperature power spectrum, which is reached for an angular size of $.5^\circ$ (approximately that subtended by the moon) if space is flat. If it has a positive curvature, spots should be larger (with a corresponding displacement of the position of the peak) and correspondingly smaller for negative curvature. The joint analysis of the considerable amount of data obtained during the last years by balloon experiments (BOOMERanG, MAXIMA, DASI) [10, 11], combined with galaxy clustering data, produced a lower bound for $|R| > 20h^{-1}$ Gpc, i.e. twice as large as the radius of the observable universe of about $R_U \simeq 9h^{-1}$ Gpc (h is the reduced Hubble constant).

3.2. On the topology

Let us repeat that GR does not prescribe the topology of the universe, or its being finite or not, and the universe could perfectly be flat and finite. The simplest non-trivial model from the theoretical viewpoint is the toroidal topology. Traces for this and more elaborated ones, as negatively curved but compact spaces, have been profusely investigated, and some circles in the sky with near identical temperature patterns were identified [12]. And yet more papers appear from time to time proposing a new topology [13]. However, to summarize all these efforts and the observational situation, and once the numerical data are interpreted without bias (what sometimes was not the case and led to erroneous conclusions), it seems at present that available data point towards a very large (we may call it *infinite*) flat space.

4. Vacuum energy fluctuations and the cosmological constant

The issue of the cc has got renewed thrust from the observational evidence of an acceleration in the expansion of our universe, initially reported by two different groups [14]. There was some controversy on the reliability of the results obtained from those observations and on its precise interpretation, by a number of different reasons. Anyway, after new data have been gathered, there is now consensus among the community of cosmologists that, in fact, an acceleration is there, and that it has the order of magnitude obtained in the above mentioned observations [15–17]. As a consequence, many theoreticians have urged to try to explain this fact, and also to try to reproduce the precise value of the cc coming from these observations [18–20].

Now, as crudely stated by Weinberg [21], it is even more difficult to explain why the cc is so small but non-zero than to build the theoretical models where it exactly vanishes [22]. Rigorous calculations performed in quantum field theory on the vacuum energy density, ρ_V, corresponding to quantum fluctuations of the fields we observe in nature, lead to values that are over 120 orders of magnitude in excess of those allowed by observations of the spacetime around us. Energy always gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress–energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$ appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu}). \qquad (2)$$

It affects cosmology: $\tilde{T}_{\mu\nu}$ contains excitations above the vacuum and is equivalent to a *cc* $\lambda = 8\pi G\mathcal{E}$. Recent observations yield [23]

$$\lambda_{\rm obs} = (2.14 \pm 0.13 \times 10^{-3}\ {\rm eV})^4 \sim 4.32 \times 10^{-9}\ {\rm erg\ cm}^{-3}.$$

It is an old idea that the cc gets contributions from zero-point fluctuations [24]:

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \qquad k = 2\pi/\lambda. \qquad (3)$$

Evaluating in a box and putting a cut-off at maximum $k_{\rm max}$ corresponding to reliable QFT physics (e.g., the Planck energy)

$$\rho \sim \frac{\hbar k_{\rm Planck}^4}{16\pi^2} \sim 10^{123} \rho_{\rm obs}. \qquad (4)$$

Assuming one will be able to prove (in the future) that the ground value of the cc is *zero* (as many suspected until recently), we will be left with this *incremental value* coming from the topology or BCs. This sort of two-step approach to the cc is becoming more and more popular recently as a way to try to solve this very difficult issue [25]. We have seen, using different examples, that this value acquires in fact the correct order of magnitude—corresponding to that coming from the observed acceleration in the expansion of our universe—under some reasonable conditions. We put forward a quite simple and primitive idea (but, for the same reason, of possibly far reaching consequences), related with the *global* topology of the universe [26] and in connection with the possibility that a faint scalar field pervading the universe could exist. Fields of this kind are ubiquitous in inflationary models, quintessence theories and the like. In other words, we do not pretend to solve the old problem of the cc, not even to contribute significantly to its understanding, but just to present simple and usual models which show that the right order of magnitude of (some contributions to) ρ_V which lie in the precise range deduced from the astrophysical observations are not difficult to get. To say it in different words, we only address here the 'second stage' of what has been termed by Weinberg [21] the *new* cc problem.

5. Vacuum energy contribution in different models

5.1. Simple model with large and small compactified dimensions

We assume the existence of a scalar field extending through the universe and calculate the contribution to the cc from the Casimir energy density of this field, for some typical boundary conditions. Ultraviolet contributions will be safely set to zero by some mechanism of a fundamental theory. Another hypothesis will be the existence of both large and small dimensions (the total number of large spatial coordinates being always three), some of which may be compactified, so that the global topology of the universe will play an important role. There is a quite extensive literature both in the subject of what is the global topology of spatial sections of the universe [26] and also on the issue of the possible contribution of the Casimir effect as a source of some sort of cosmic energy, as in the case of the creation of a neutron star [27]. There are arguments that favour different topologies, as a compact hyperbolic manifold for the spatial section, what would have clear observational consequences [28]. Other interesting work along these lines was reported in [29] and related ideas have been discussed very recently in [30]. However, we differ from all those in that emphasis is put now in obtaining the right order of magnitude for the effect. At the present stage it has no sense to consider the whole amount of possibilities concerning the nature of the field, the different models for the topology of the universe, and the different BCs possible, with its effect on the sign of the force too. This is left to a second, more detailed analysis. From the previous results [3] we know that the range of orders of magnitude of the vacuum energy density for the most common possibilities is not so widespread and may only differ by at most a couple of digits. This will allow us, both for the sake of simplicity and universality, to deal with two simple situations, corresponding to a scalar field with periodic BCs, or spherically compactified. As explained in [31], most cases with usual BCs reduce to those from a mathematical viewpoint.

For the lack of space we will not describe this model in detail here (this has been done elsewhere [32]). Suffice to say that it can be proven that the contribution of the vacuum energy of a small-mass scalar field, conformally coupled to gravity, and coming from the compactification of some small (2 or 3) and some large (1 or 2) dimensions—with compactification radii of the order of 10–1000 the Planck length in the first case and of the order of the present radius of the universe, in the second—leads to values that compare well with observational data, in order of magnitude, with the exception of the *sign* —which turns out to be opposite to that needed to explain negative pressure. To deal with this crucial issue, we consider the two following classes of models.

5.2. Braneworld models

Braneworld theories may hopefully solve both the hierarchy problem and the cc problem. The bulk Casimir effect can play an important role in the construction (radion stabilization) of braneworlds. We have calculated the bulk Casimir effect (effective potential) for conformal and for massive scalar fields [33]. The bulk is a five-dimensional AdS or dS space, with two (or one) four-dimensional dS branes (our universe). The results obtained are consistent with observational data.

For the case of two dS_4 branes (at L separation) in a dS_5 background (it becomes a one-brane configuration as $L \to \infty$), for the Casimir energy density and effective potential, for a conformally invariant scalar-gravitational theory $\mathcal{S} = \frac{1}{2}\int d^5x\,\sqrt{g}[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \xi_5 R^{(5)}\phi^2]$, $\xi_5 = -3/16$, with $R^{(5)}$ the curvature and $ds^2 = g_{\mu\nu}dx^\mu\,dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right)$ the Euclidean metric of the five-dimensional AdS bulk, $d\Omega_4^2 = d\xi^2 + \sin^2\xi\, d\Omega_3^2$—with α being the AdS radius, related to the cc of the AdS bulk and $d\Omega_3$ the metric on the 3-sphere— we obtain what follows. For the one-brane Casimir energy density (pressure), we get

$$\mathcal{E}_{\mathrm{Cas}} = \frac{\hbar c}{2LR^4}\zeta\left(-\frac{1}{2}\middle| L_5\right) = -\frac{\hbar c\pi^3}{36L^6}\left[\frac{\pi^2}{315} - \frac{1}{240}\left(\frac{L}{\mathcal{R}}\right)^2 + \mathcal{O}\left(\frac{L}{\mathcal{R}}\right)^4\right], \quad (5)$$

which is about ten times larger than the ordinary Casimir effect: $\mathcal{E}_{\mathrm{CE}} = -\frac{\hbar c\pi^2}{240L^4}$ (about 100 dynes cm^{-2} at 100 nm, although in different dimensions, so that the comparison should be taken with care). For the one-loop effective potential, we obtain $V = \frac{1}{2L\,\mathrm{Vol}(M_4)}\log\det(L_5/\mu^2)$, where $L_5 = -\partial_z^2 - \Delta^{(4)} - \xi_5 R^{(4)} = L_1 + L_4$, and $\log\det L_5 = \sum_{n,\alpha}\log\left(\lambda_n^2 + \lambda_\alpha^2\right) = -\zeta'(0|L_5)$. In the one-brane limit $L \to \infty$, $K_t(L_1) \sim \frac{L}{2\sqrt{\pi t}}$ and $\zeta'(0|L_5) = \frac{1}{3\mathcal{R}}\left[\zeta_H\left(-4, \frac{3}{2}\right) - \frac{1}{4}\zeta_H\left(-2, \frac{3}{2}\right)\right] = 0$. And the small distance expansion for the effective potential yields (up to an overall factor)

$$\begin{aligned}
\zeta'(0|L_5) &= \frac{\zeta'(-4)}{6}\frac{\pi^4\mathcal{R}^4}{L^4} + \frac{\zeta'(-2)}{12}\frac{\pi^2\mathcal{R}^2}{L^2} + \frac{1}{24}\left[\zeta_H'(-4, 3/2) - \frac{1}{2}\zeta_H'(-2, 3/2)\right]\ln\frac{\pi^2\mathcal{R}^2}{L^2} \\
&\quad + \frac{\zeta'(0)}{6}\left[\zeta_H'(-4, 3/2) - \frac{1}{2}\zeta_H'(-2, 3/2)\right] + \frac{1}{24}\zeta_H'(-4, 3/2) \\
&\quad + \frac{1}{36}\left[\frac{1}{8}\zeta_H'(-4, 3/2) - \frac{1}{3}\zeta_H'(-6, 3/2)\right]\frac{L^2}{\mathcal{R}^2} + \mathcal{O}\left(\frac{L^4}{\pi^4\mathcal{R}^4}\right) \\
&\simeq 0.129\,652\frac{\mathcal{R}^4}{L^4} - 0.025\,039\frac{\mathcal{R}^2}{L^2} - 0.002\,951\ln\frac{\mathcal{R}^2}{L^2} \\
&\quad - 0.017\,956 - 0.000\,315\frac{L^2}{\mathcal{R}^2} + \cdots. \qquad (6)
\end{aligned}$$

On the other hand, the effective potential for the massive scalar field model is obtained to be $V = \frac{1}{2L\,\mathrm{Vol}(M_4)}\log\det(L_5/\mu^2)$, $L_5 \equiv -\partial_z^2 + m^2l^2\sinh^{-2}z - \Delta^{(4)} - \xi_5 R^{(4)} = L_1 + L_4$ (AdS), $L_5 \equiv -\partial_z^2 + m^2\cosh^{-2}z - \Delta^{(4)} - \xi_5 R^{(4)} = L_1 + L_4$ (dS). For the small mass limit (with L not large), it yields

$$\zeta'(0|L_5) \simeq \frac{a\rho + a^2\rho^2}{48} - \frac{\pi^2}{144}\left\{\frac{a\rho^2}{2} + \left[2\zeta'(-4,3/2) - \zeta'(-2,3/2)\right]\rho\right\} - \frac{\pi^4}{4370}[2\zeta'(-4,3/2) - \zeta'(-2,3/2)]\rho^2 + \mathcal{O}(m^6),$$
$$a \equiv \frac{\pi^2\mathcal{R}^2}{L^2}, \qquad \rho \equiv \frac{m^2l^2}{\pi^2}\frac{\tanh(L/2l)}{L/2l}, \tag{7}$$

while for the large mass limit (with L not small), it is

$$\zeta'(0|L_5) = -\frac{4m^2l^3}{3\mathcal{R}}\frac{\arctan(\sinh L/2l)}{\sinh(L/2l)} + \cdots, \tag{8}$$

which is now non-zero (unlike in previous calculations, which turned a vanishing value) and can fit the observed order of magnitude under appropriate conditions.

5.3. Supergraviton theories

Finally, we have also computed the effective potential for some multi-graviton models with supersymmetry [34]. In one case, the bulk is a flat manifold with the torus topology $\mathbf{R}\times\mathbf{T}^3$, and it can be shown that the induced cc can be rendered *positive* due to topological contributions [35]. Previously, the case of $\mathbf{R}^4$ had been considered. In the multi-graviton model the induced cc can indeed be positive, but only if the number of massive gravitons is sufficiently large, what is not easy to fit in a natural way. In the supersymmetric case, however, the cc turns out to be positive just by imposing anti-periodic BC in the fermionic sector. An essential issue in our model is to allow for non-nearest-neighbour couplings.

The multi-graviton model is defined by taking N copies of the fields with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$ and φ_n. Our theory is defined by a Lagrangian which is a generalization of that in [36]. It reads

$$\mathcal{L} = \sum_{n=0}^{N-1}\Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h^\lambda_{n\mu}\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n\partial^\lambda h_n - \frac{1}{2}\left(m^2\Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu\varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right) - \frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]. \tag{9}$$

Δ and $\Delta^\dagger$ are difference operators, which operate on the indices n as $\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n+k}$, $\Delta^\dagger\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n-k}$, $\sum_{k=0}^{N-1} a_k = 0$, where a_k are the N constants and the N variables ϕ_n can be identified with periodic fields on a lattice with N sites if the periodic boundary conditions, $\phi_{n+N} = \phi_n$ are imposed. The latter condition assures that Δ becomes the usual differentiation operator in a properly defined continuum limit.

In the case when anti-periodic boundary conditions are imposed in the fermionic sector, the situation changes completely with respect to the bosonic one, since the fermionic mass spectrum becomes quite different. The one-loop effective potential in the anti-periodic case is calculated to be

$$V_{\rm eff} = \frac{M_1^4}{4\pi^2}\left(\ln\frac{M_1^2}{\mu_R^2} - \frac{3}{2}\right) - \frac{4M_1^4}{3\pi^2}\int_1^\infty \mathrm{d}u\, G(M_1ru)(u^2-1)^{3/2} - \frac{\tilde{M}_0^4}{4\pi^2}\left(\ln\frac{\tilde{M}_0^2}{\mu_R^2} - \frac{3}{2}\right) + \frac{4\tilde{M}_0^4}{3\pi^2}\int_1^\infty \mathrm{d}u\, G(\tilde{M}_0ru)(u^2-1)^{3/2} - \frac{\tilde{M}_1^4}{8\pi^2}\left(\ln\frac{\tilde{M}_1^2}{\mu_R^2} - \frac{3}{2}\right)$$

$$+\frac{2\tilde{M}_1^4}{3\pi^2}\int_1^\infty \mathrm{d}u\, G(\tilde{M}_1 r u)(u^2-1)^{3/2} = -\frac{m^4}{36\pi^2}\log\frac{2^{16}}{3^9} + V_T, \tag{10}$$

where V_T is the sum of all the topological contributions. Note that the first term on the rhs is always negative, but the whole effective potential can be positive, due to the presence of the topological term. Thus, in the regime $mr \ll 1$ one has $V_T \sim 1/8\pi^2 r^4$, that is, $V_{\mathrm{eff}} > 0$ for $mr < [(2/9)\log(2^{16}3^{-9})]^{-1/4} \sim 1.4$, while in the opposite regime, $mr \gg 1$; we can see that the topological contribution (although still positive) is negligible, and the effective potential remains negative. A change of sign in the correct region is obtained.

For the torus topology we have got the topological contributions to the effective potential to have always a fixed sign, which depends on that BC imposes. They are negative for periodic fields and positive for anti-periodic fields. But topology provides then a mechanism which, in a natural way, permits to have a positive cc in the multi-supergravity model with anti-periodic fermions. The value of the cc is regulated by the corresponding size of the torus. We can most naturally use the minimum number, $N = 3$, of copies of bosons and fermions, and show that—as in the first, much more simple example, but now with the right sign!— within our model the observational values for the cc can be matched, by making quite reasonable adjustments of the parameters involved. As a byproduct, the results that we have obtained [35] may also be relevant in the study of electroweak symmetry breaking in models with similar type of couplings, for the deconstruction issue.

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References

[1] Collins J 1984 *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion* (Cambridge: Cambridge University Press)
McComb W D 2004 *Renormalization Methods: A Guide for Beginners* (Oxford: Oxford University Press)
[2] Haro J and Elizalde E 2006 All time physical approach to the dynamical Casimir effect *Phys. Lett. Rev.* (to appear) (*Preprint* hep-th/0607186)
[3] Elizalde E, Odintsov S D, Romeo A, Bytsenko A A and Zerbini S 1994 *Zeta Regularization Techniques with Applications* (Singapore: World Scientific)
Elizalde E 1995 *Ten Physical Applications of Spectral Zeta Functions* (Berlin: Springer)
Kirsten K 2001 *Spectral Functions in Mathematics and Physics* (London: Chapman & Hall)
[4] Elizalde E 1999 *J. High Energy Phys.* JHEP07(1999)015
[5] Deutsch D and Candelas P 1979 *Phys. Rev.* D **20** 3063
[6] Symanzik K 1981 *Nucl. Phys.* B **190** 1
[7] Jaffe R L 2003 Unnatural acts: unphysical consequences of imposing boundary conditions on quantum fields *Preprint* hep-th/0307014
Graham N, Jaffe R L, Khemani V, Quandt M, Scandurra M and Weigel H 2003 *Phys. Lett.* B **572** 196
Graham N, Jaffe R L, Khemani V, Quandt M, Scandurra M and Weigel H 2002 *Nucl. Phys.* B **645** 49
Graham N, Jaffe R L and Weigel H 2002 *Int. J. Mod. Phys.* A **17** 846
[8] Mostepanenko V M and Trunov N N 1997 *The Casimir Effect and its Application* (Oxford: Clarendon)
Milton K A 2001 *The Casimir Effect: Physical Manifestations of Zero-Point Energy* (Singapore: World Scientific)
Bordag M, Mohideen U and Mostepanenko V M 2001 *Phys. Rep.* **353** 1
[9] Tegmark M 2002 *Science* **296** 1427
[10] de Bernardis P *et al* 2000 *Nature* **404** 955

[11] Hanany S *et al* 2000 *Astrophys. J.* **545** L5
Balbi A *et al* 2000 *Astrophys. J.* **545** L1
Balbi A *et al* 2001 *Astrophys. J.* **558** L145 (erratum)
[12] Cornish N J *et al* 2004 *Phys. Rev. Lett.* **92** 201302
[13] Luminet J-P *et al* 2003 *Nature* **425** 593
[14] Perlmutter S *et al* (Supernova Cosmology Project Collaboration) 1999 *Astrophys. J.* **517** 565
Riess A G *et al* (Hi-Z Supernova Team Collaboration) 1998 *Astron. J.* **116** 1009
[15] Riess A G 2000 *Publ. Astron. Soc. Pac.* **112** 1284
[16] Carroll S M 2001 *Living Rev. Rel.* **4** 1
[17] Carroll S M 2003 Why is the Universe accelerating? *Measuring and Modeling the Universe (Carnegie Observatories Astrophysics Series* vol 2*)* ed W L Freedman (Cambridge: Cambridge University Press) (*Preprint* astro-ph/0310342)
[18] Sahni V and Starobinsky A 2000 *Int. J. Mod. Phys.* D **9** 373
[19] Shapiro I L and Solà J 2000 *Phys. Lett.* B **475** 236
[20] Mongan T R 2001 *Gen. Rel. Grav.* **33** 1415
[21] Weinberg S 2000 *Phys. Rev.* D **61** 103505
[22] Weinberg S 1989 *Rev. Mod. Phys.* **61** 1
Fischler W, Klebanov I, Polchinski J and Susskind L 1989 *Nucl. Phys.* B **237** 157
Coleman S 1988 *Nucl. Phys.* B **310** 643
Coleman S 1988 *Nucl. Phys.* B **307** 867
Weinberg S 1987 *Phys. Rev. Lett.* **59** 2607
Baum E 1984 *Phys. Lett.* B **133** 185
Hawking S W 1995 *Shelter Island II: Proc. 1983 Shelter Island Conf. on Quantum Field Theory and the Fundamental Problems of Physics* ed R Jackiw *et al* (Cambridge: MIT Press)
Hawking S W 1984 *Phys. Lett.* B **134** 403
[23] Tegmark M *et al* (SDSS Collaboration) 2004 *Phys. Rev.* D **69** 103501
Eisenstein D J *et al* 2005 *Astrophys. J.* **633** 560
Adelman-McCarthy J K *et al* 2006 *Astrophys. J. Suppl.* **162** 38
[24] Zeldovich Ya B 1968 *Sov. Phys. Usp.* **11** 382
[25] Burgess C P *et al* 2006 *Preprints* hep-th/0606020, 0510123
Padmanabhan T 2006 *Preprint* gr-qc/0606061
Parker L and Raval A 2000 *Phys. Rev.* D **62** 083503
Parker L and Raval A 2001 *Phys. Rev. Lett.* **86** 749
[26] Blanloeil V and Roukema B F (ed) 2000 *Cosmological Topology in Paris 1998* (*Preprint* astro-ph/0010170) (see also the entire vol 15 of *Classical and Quantum Gravity* (1998))
[27] Sokolov I Yu 1993 *JETP Lett.* **57** 617
[28] Cornish N J, Spergel D and Starkman G 1998 *Class. Quantum Grav.* **15** 2657
Cornish N J, Spergel D and Starkman G 1996 *Phys. Rev. Lett.* **77** 215
Müller D, Fagundes H V and Opher R 2001 *Phys. Rev.* D **63** 123508
[29] Elizalde E 1994 *J. Math. Phys.* **35** 6100
Elizalde E 1994 *J. Math. Phys.* **35** 3308
[30] Banks T, Dine M and Nelson A E 1999 *J. High Energy Phys.* JHEP06(1999)014
[31] Elizalde E 1989 *Nuovo Cimento* B **104** 685
[32] Elizalde E 2006 *J. Phys. A: Math. Gen.* **39** 6299
[33] Elizalde E, Nojiri S, Odintsov S D and Ogushi S 2003 *Phys. Rev.* D **67** 063515
Elizalde E, Nojiri S and Odintsov S D 2004 *Phys. Rev.* D **70** 043539
[34] Boulanger N, Damour T, Gualtieri L and Henneaux M 2001 *Nucl. Phys.* B **597** 127
Sugamoto A 2003 *Grav. Cosmol.* **9** 91
Arkani-Hamed N, Cohen A G and Georgi H 2001 *Phys. Rev. Lett.* **86** 4757
Arkani-Hamed N, Georgi H and Schwartz M D 2003 *Ann. Phys., NY* **305** 96
[35] Cognola G, Elizalde E and Zerbini S 2005 *Phys. Lett.* B **624** 70
Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2004 *Mod. Phys. Lett.* A **19** 1435
[36] Kan N and Shiraishi K 2003 *Class. Quantum Grav.* **20** 4965